General Certificate of Education
Advanced Level Examination
June 2013

Mathematics MPC3

Unit Pure Core 3

Thursday 6 June 2013 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

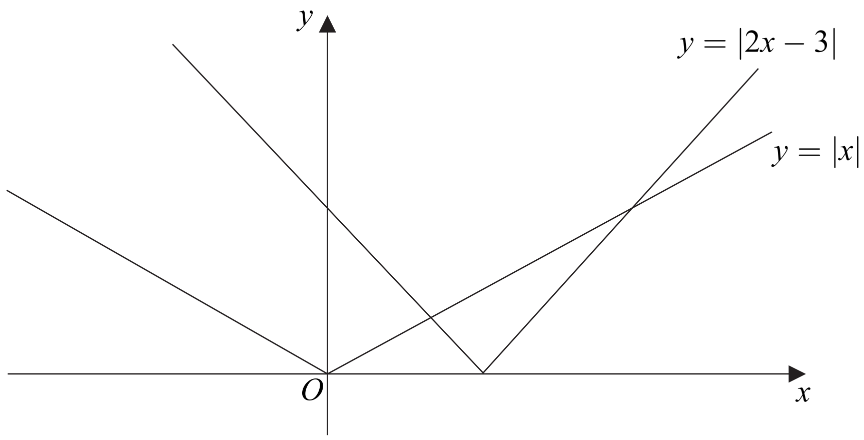
Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

1 The diagram below shows the graphs of $y = |2x - 3|$ and $y = |x|$.



(a) Find the x-coordinates of the points of intersection of the graphs of $y = |2x - 3|$ and $y = |x|$. *(3 marks)*

(b) Hence, or otherwise, solve the inequality

$$|2x - 3| \geqslant |x|$$ *(2 marks)*

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2 (a) Given that $y = x^4 \tan 2x$, find $\dfrac{dy}{dx}$. *(3 marks)*

(b) Find the gradient of the curve with equation $y = \dfrac{x^2}{x - 1}$ at the point where $x = 3$. *(3 marks)*

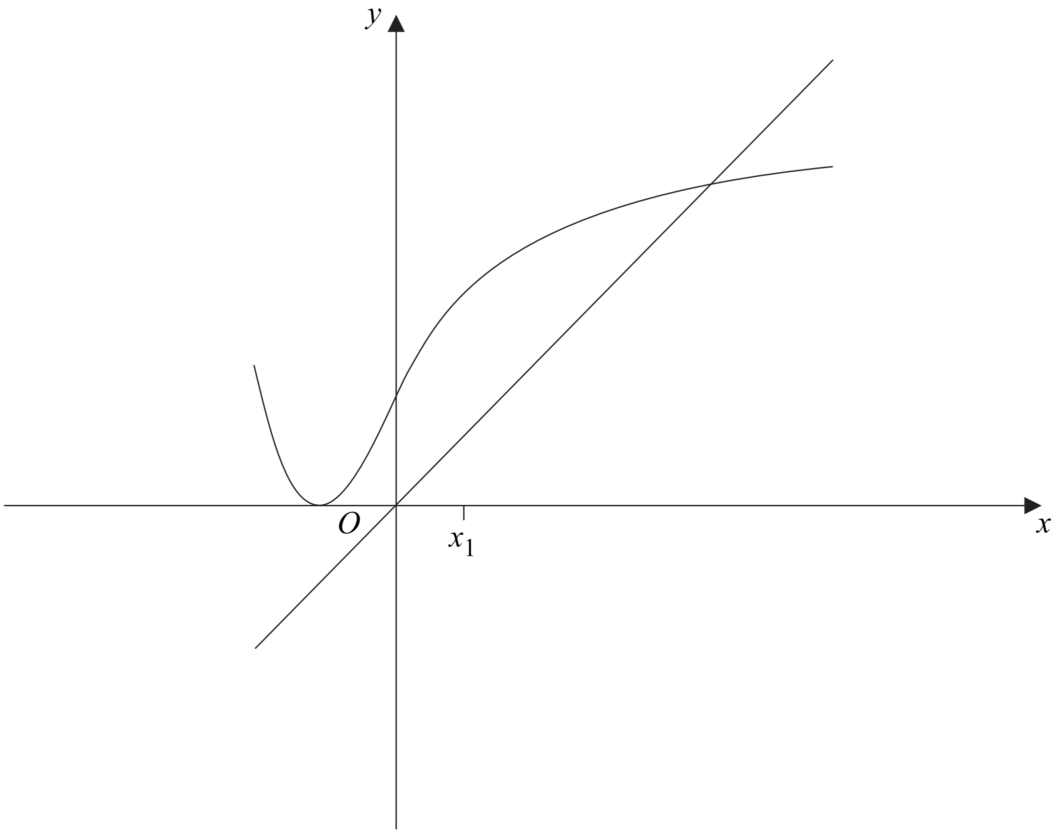
3 (a) The equation $e^{-x} - 2 + \sqrt{x} = 0$ has a single root, α.

Show that α lies between 3 and 4. *(2 marks)*

(b) Use the recurrence relation $x_{n+1} = (2 - e^{-x_n})^2$, with $x_1 = 3.5$, to find x_2 and x_3, giving your answers to three decimal places. *(2 marks)*

(c) The diagram below shows parts of the graphs of $y = (2 - e^{-x})^2$ and $y = x$, and a position of x_1.

On the diagram, draw a staircase or cobweb diagram to show how convergence takes place, indicating the positions of x_2 and x_3 on the x-axis.



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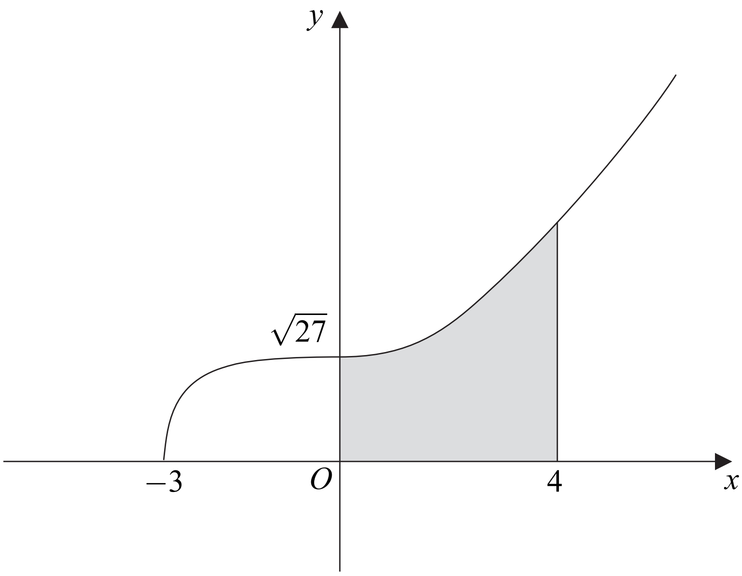
4 By forming and solving a quadratic equation, solve the equation

$$8 \sec x - 2 \sec^2 x = \tan^2 x - 2$$

in the interval $0 < x < 2\pi$, giving the values of x in radians to three significant figures. *(7 marks)*

5 The diagram shows a sketch of the graph of $y = \sqrt{27 + x^3}$.



(a) The area of the shaded region, bounded by the curve, the x-axis and the lines $x = 0$ and $x = 4$, is given by $\int_0^4 \sqrt{27 + x^3}\, dx$.

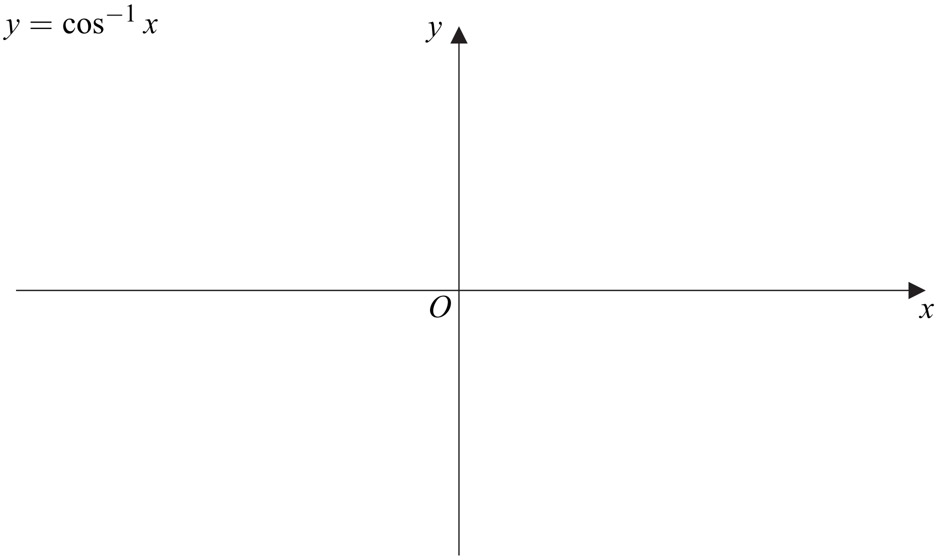
Use the mid-ordinate rule with **five** strips to find an estimate for this area. Give your answer to three significant figures. *(4 marks)*

(b) With the aid of a diagram, explain whether the mid-ordinate rule applied in part **(a)** gives an estimate which is smaller than or greater than the area of the shaded region. *(2 marks)*

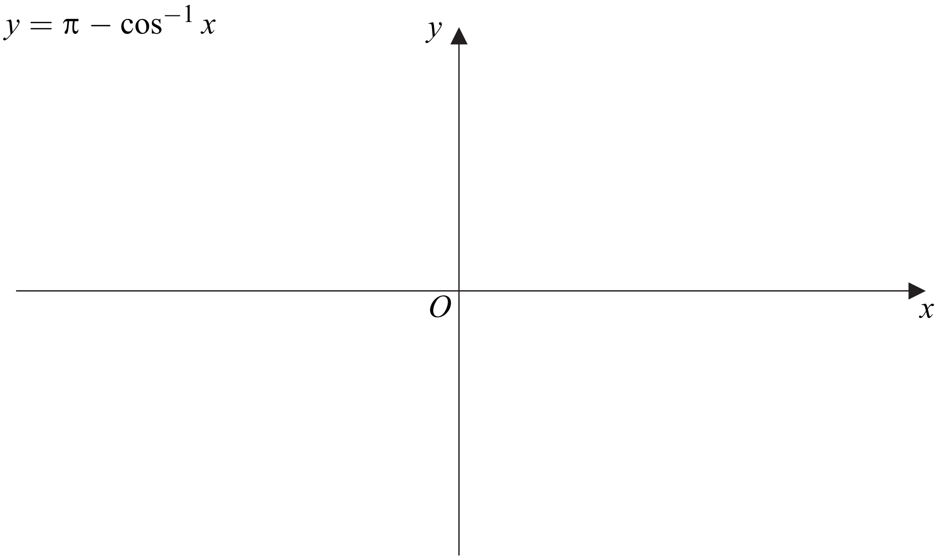
6 (a) Sketch the graph of $y = \cos^{-1} x$, where y is in radians. State the coordinates of the end points of the graph. *(2 marks)*

(b) Sketch the graph of $y = \pi - \cos^{-1} x$, where y is in radians. State the coordinates of the end points of the graph. *(2 marks)*

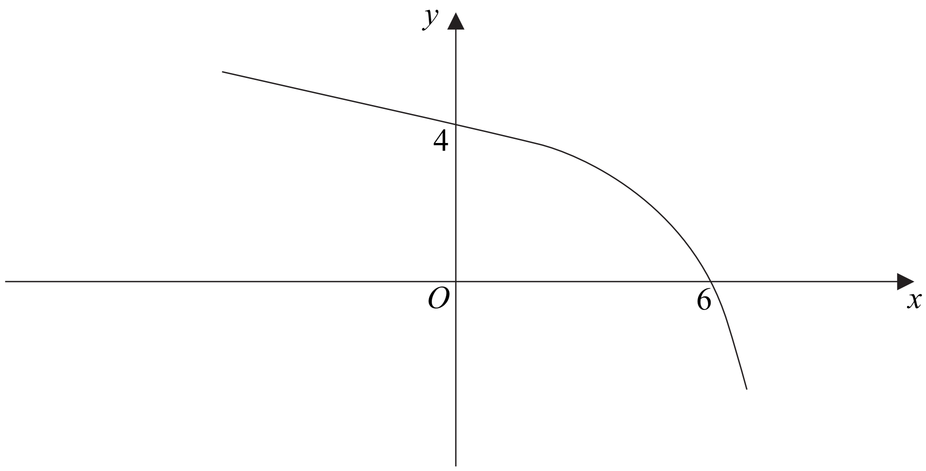
$y = \cos^{-1} x$



$y = \pi - \cos^{-1} x$



7 The diagram shows a sketch of the curve with equation $y = \mathrm{f}(x)$.



(a) On **Figure 1**, below, sketch the curve with equation $y = -\mathrm{f}(3x)$, indicating the values where the curve cuts the coordinate axes. *(2 marks)*

(b) On **Figure 2**, on page 7, sketch the curve with equation $y = \mathrm{f}(|x|)$, indicating the values where the curve cuts the coordinate axes. *(3 marks)*

(c) Describe a sequence of two geometrical transformations that maps the graph of $y = \mathrm{f}(x)$ onto the graph of $y = \mathrm{f}\left(-\frac{1}{2}x\right)$. *(4 marks)*

Figure 1

$y = -\mathrm{f}(3x)$

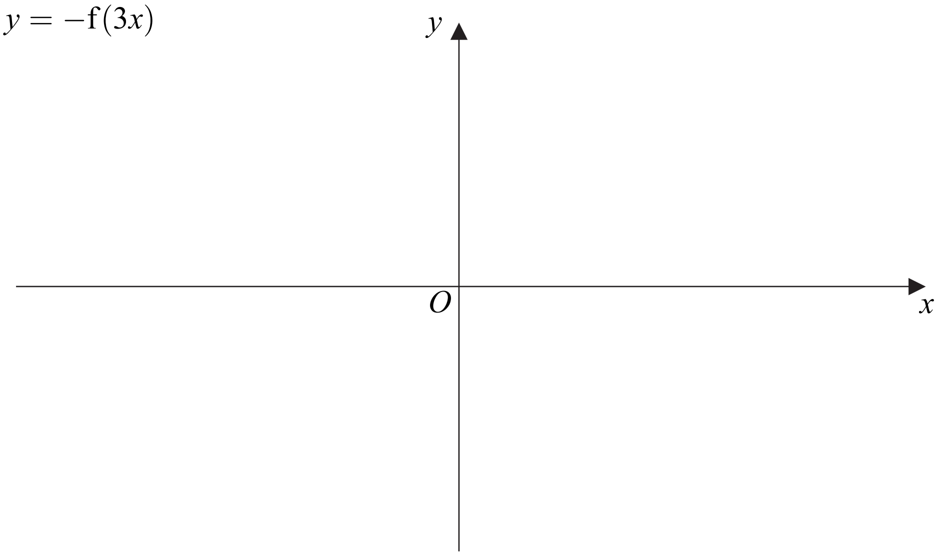
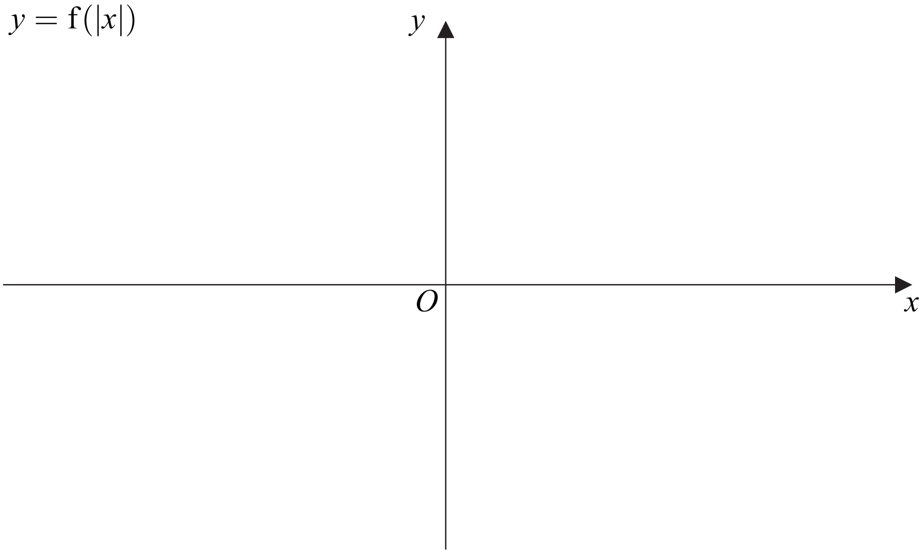
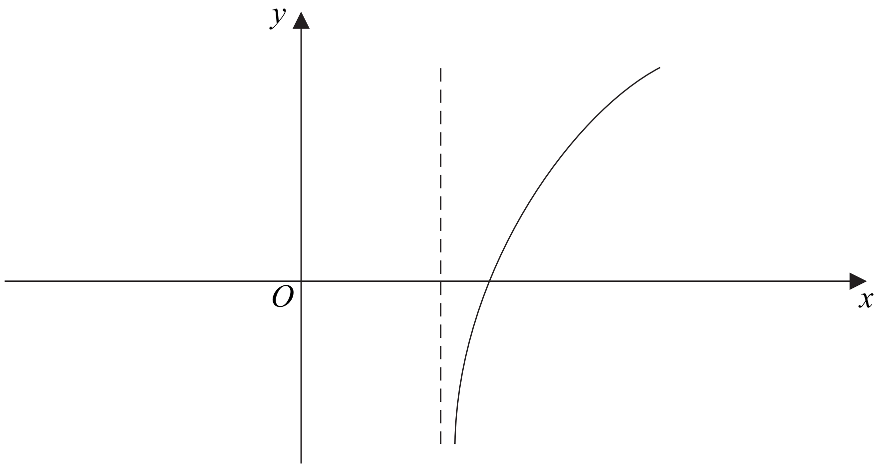


Figure 2

$y = \mathrm{f}(|x|)$



8 The curve with equation $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \ln(2x - 3)$, $x > \frac{3}{2}$, is sketched below.



(a) The inverse of f is f^{-1}.

(i) Find $\mathrm{f}^{-1}(x)$. *(3 marks)*

(ii) State the range of f^{-1}. *(1 mark)*

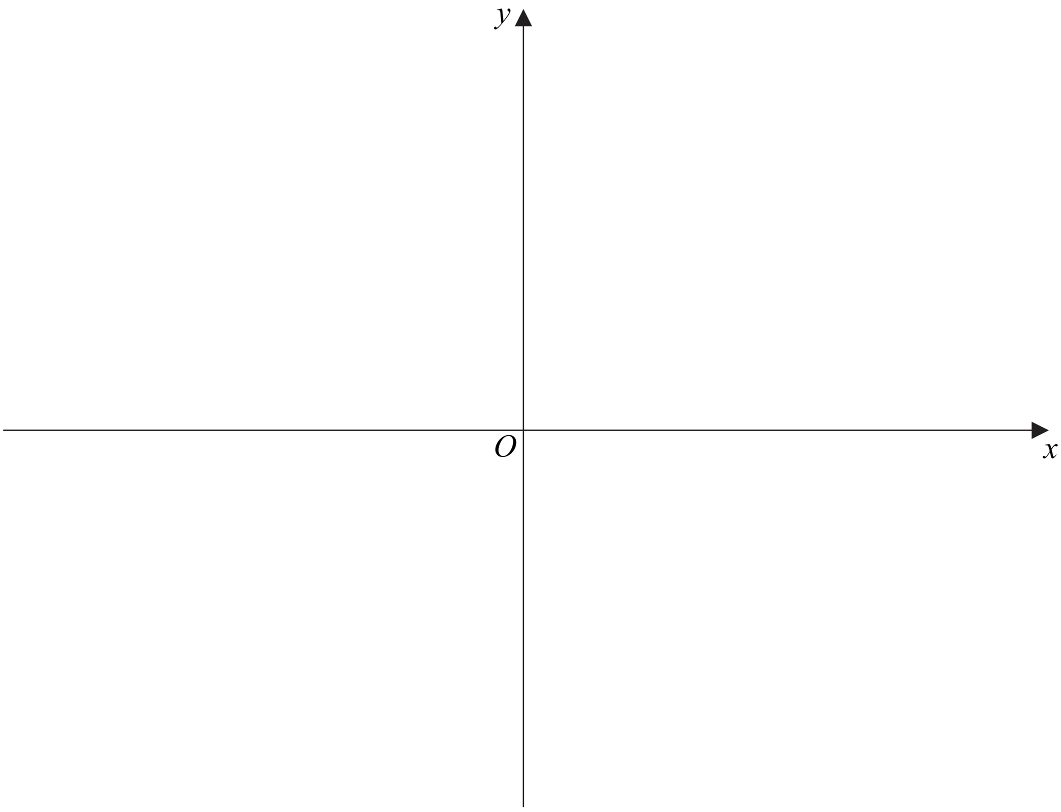
(iii) Sketch, on the axes given on page 9, the curve with equation $y = \mathrm{f}^{-1}(x)$, indicating the value of the y-coordinate of the point where the curve intersects the y-axis. *(2 marks)*

(b) The function g is defined by

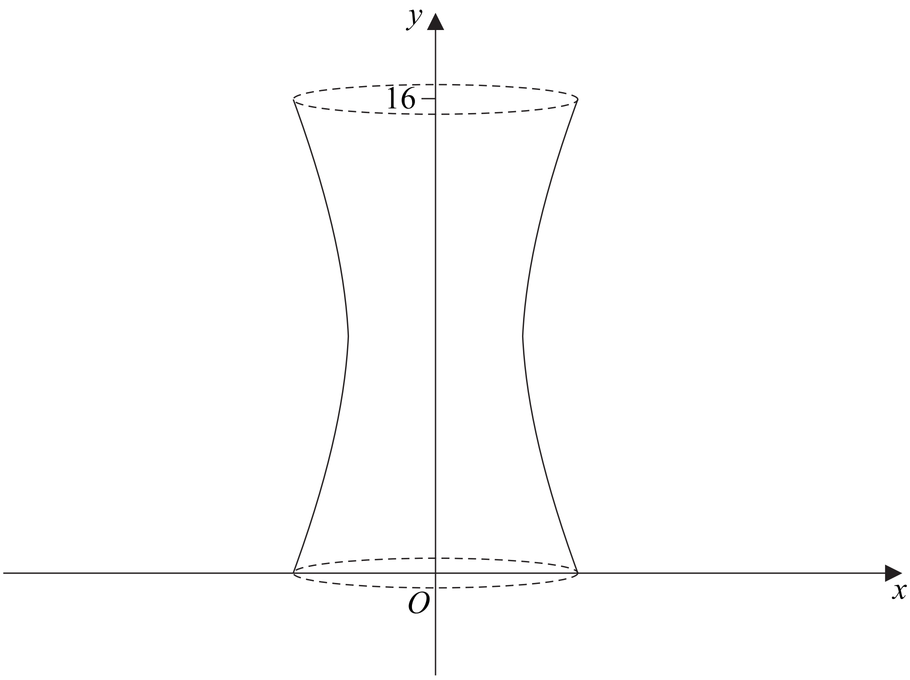
$$\mathrm{g}(x) = \mathrm{e}^{2x} - 4, \quad \text{for all real values of } x$$

(i) Find $\mathrm{gf}(x)$, giving your answer in the form $(ax - b)^2 - c$, where a, b and c are integers. *(3 marks)*

(ii) Write down an expression for $\mathrm{fg}(x)$, and hence find the exact solution of the equation $\mathrm{fg}(x) = \ln 5$. *(3 marks)*



9 The shape of a vase can be modelled by rotating the curve with equation $16x^2 - (y-8)^2 = 32$ between $y = 0$ and $y = 16$ completely **about the *y*-axis**.



The vase has a base.

Find the volume of water needed to fill the vase, giving your answer as an exact value. *(5 marks)*

10 (a) (i) By writing $\ln x$ as $(\ln x) \times 1$, use integration by parts to find $\int \ln x \, dx$. *(4 marks)*

(ii) Find $\int (\ln x)^2 \, dx$. *(4 marks)*

(b) Use the substitution $u = \sqrt{x}$ to find the exact value of

$$\int_1^4 \frac{1}{x + \sqrt{x}} \, dx$$ *(7 marks)*